Nonminimal Global Monopoles and Bound Orbits

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Abstract

We perform a numerical analysis of the gravitational field of a global monopole coupled nonminimally to gravity, and find that, for some given nonminimal couplings (in contrast with the minimal coupling case), there is an attractive region where bound orbits exist. We exhibit the behavior of the frequency shifts that would be associated with "rotation curves" of stars in circular orbits in the spacetimes of such global monopoles.

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An interesting property of global monopoles is that they are configurations with energy density decreasing with the distance as r^{-2} [1]. This is very suggestive in view of the fact that, naively at least, this is precisely what seems to be required to provide a natural explanation for the flatness of the rotation curves in spiral galaxies. Moreover by assuming the existence of a global monopole in a typical galaxy the total Newtonian mass contribution of the portion of the global monopole contained within r_{gal} is found to be $M \approx \alpha r_{gal}/2$, where $\alpha = 8\pi G\eta^2$ is the deficit angle of the global monopole spacetime. If we take the radial extent of a galaxy to be $r_{gal} \approx 15\,\mathrm{Kpc}$ and consider a typical grand unified theory with a symmetry breaking scale $\eta \approx 10^{16}$ GeV, this mass turns out to be $\approx 10^{69}$ GeV which is ten times the total mass due to the 10^{11} solar-mass stars in the typical galaxy: $M_{stars} \approx 10^{68}$ GeV. This again seems to be precisely what is called for in the observations [2]. Finally, and what seems to be an added bonus, if we assume that the field of the monopole extends on average a distance of 10 galactic radii from the galaxy, where the configuration presumably coincides with that of the monopole centered in the neighboring galaxy, the total mass associated with each monopole (and thus with each galaxy) turns out to be 100 times that of the galaxy. Thus the total contribution of the monopoles to the average density in the universe is of the right order of magnitude to account for the inflationary prediction of a universe with critical density. This is in fact the basic argument that allows one to place bounds on the number density of global monopoles present in the universe [3], the exact nature of which depends of course on the value taken for η. Indeed global monopoles have been shown to be able to trigger inflation on their own for suitably high η [4], [5], [6].

Unfortunately this picture doesn't hold up to scrutiny at the next level of analysis because of two problems: 1) upon substitution of Newtonian gravity by general relativity, it turns out [1], [7] that the linearly divergent mass has, at large distances, an effect analogous to that of a deficit solid angle plus that of a tiny mass at the origin. The mass of this "monopole core" is about $0.8\,\alpha$. In fact Harari and Loustó [7] showed that this small core mass is negative and produces a repulsive potential. They studied the motion of test particles, in the large distance region, in the spacetime of a global monopole, concluding that there are no bound orbits; 2) the fact that the monopole configuration is rather unique, in the sense that it is basically independent of the ordinary matter content in the corresponding galaxy, which conflicts with the fact that there is a rather large range of galactic masses for which the dark matter component is

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about 10 times more massive than the ordinary matter component [2].

In spite of these remarks the initially mentioned results are in our view still very suggestive, specially if we look at the remarkable degree of universality that seems to emerge from the systematic study of galactic dynamics [8]: 1) the ordinary matter content is such a good indicator of the mass of the dark matter component; 2) the universality of the form of the radial distribution of the dark matter content; 3) other seemingly coincidental facts as scaling properties between dark and luminous galactic structure parameters. We, therefore, believe that the general idea of monopoles as seeds of galaxy formation, and models for the galactic (and perhaps cosmological) dark matter deserve to be further explored.

The purpose of this Letter is to illustrate the possibilities in these models by considering the simplest nontrivial modification of the global monopole picture: nonminimally coupled global monopoles. In this work we will show that the situation regarding the problems alluded to above, can change dramatically with the introduction of a nonminimal coupling.

Let's consider the simplest such possibility, namely a theory of a triplet of scalar fields ϕ^a, $a = 1, 2, 3$, nonminimally coupled (NMC) to gravity with global O(3) symmetry which is broken spontaneously to U(1). The Lagrangian for the model is

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{16\pi}R + F(R, \phi^a\phi^a)\right) - \sqrt{-g}\left[\frac{1}{2}(\nabla\phi^a)^2 + V(\phi^a\phi^a)\right] \quad . \tag{1}$$

where $V(\phi^a\phi^a)$ is a potential that results in the breaking of the symmetry and is usually taken to be the Mexican hat potential $V(\phi^a\phi^a) = \frac{\lambda}{4}(\phi^a\phi^a - \eta^2)^2$, and F is an arbitrary function of two variables. Note that in these theories the resulting potential for the scalar fields (the terms in the Lagrangian that are independent of the scalar field gradients) is

$$V(\phi^a\phi^a)_{res} = V(\phi^a\phi^a) - F(R, \phi^a\phi^a) \quad . \tag{2}$$

Therefore, the ordinary matter would affect the location of the minimum of the effective potential through its effect on R, thus avoiding the scenario where the monopole configuration is universal. This opens the possibility to recover the correlation between the masses in the dark and ordinary matter components of the galaxy that were alluded before.

Next we focus on the existence or lack of existence of bound orbits in the corresponding spacetimes. To do this we will look at a specific example corresponding to the choice $F = (\xi\phi^a\phi^a)R$ (where ξ is a constant) and proceed to construct the resulting spacetime and analyze the motion of test particles. We will specialize to the case of a spherical and static solution to the coupled field equations. The configuration describing a monopole is taken as usual

$$\phi^a = \eta f(r)\frac{x^a}{r}, \tag{3}$$

with $x^a x^a = r^2$, so that we will actually have a monopole solution if $f \to 1$ at spatial infinity. We adopt the following metric:

$$ds^2 = -N^2(r)dt^2 + A^2(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \quad , \tag{4}$$

and analyze solutions of the gravitational and scalar fields equations describing global monopole configurations and the resulting space-time. Owing to the complexity of the resulting equations, we will perform a numerical analysis. To do so we adopt the following variables:

$$\nu(\tilde{r}) = \ln[N(\tilde{r})] \quad , \tag{5}$$

$$A(\tilde{r}) = \left(1 - \alpha - \frac{2m(\tilde{r})}{\tilde{r}}\right)^{-1/2} \quad , \tag{6}$$

where

$$\alpha = \frac{\Delta}{1 + 2\xi\Delta}, \quad \Delta = 8\pi\eta^2 \quad , \tag{7}$$

where we have introduced the dimensionless quantity $\tilde{r} \equiv r/r_c \equiv \eta\lambda^{1/2} \cdot r$.

The solutions are obtained by the standard shooting method [9] with boundary conditions ensuring a regular origin, i.e., $f(0) = 0$, $m(0) = 0$, $m(0)_{,\tilde{r}} = -\alpha/2$, and that the spacetime is asymptotically flat but for a deficit angle [10], i.e., $f(\infty) = 1$, $\nu(\infty) = \frac{1}{2}\ln[1 - \alpha]$. The "shooting parameter" is taken to be $f(0)_{,\tilde{r}}$.

Then we calculate the ADM (gravitational) mass of these configurations, (see [10] for a rigorous definition of the ADM mass for spacetimes that are asymptotically flat but for a deficit angle) which can be easily evaluated from the integral

$$M_{\mathrm{ADM}\alpha} = \lim_{\tilde{r}\to\infty} m(\tilde{r}) = \int_0^\infty \left(4\pi\tilde{r}^2 T_t^t(\tilde{r}) - \frac{\alpha}{2}\right) d\tilde{r} \quad . \tag{8}$$

For all values of ξ that we analyzed the resulting ADM mass was negative as in the minimal coupling case (here we must remind the reader that the ADM mass for quasi-asymptotically flat spacetimes does not share the positiveness property of the ADM mass of asymptotically flat spacetimes). For $\xi = -2$, for example (corresponding to the configuration we will analyze in more detail) the resulting ADM mass is $\approx -2.5\,\alpha$.

We now study the motion of test particles around a global monopole. To do this we consider the "equatorial geodesics" for the metric (4) (i.e., geodesics lying on the plane $\theta = \pi/2$). We denote the 4-velocity of a test particle as $u^\mu = (\dot{t}, \dot{r}, 0, \dot{\varphi})$, where the dot stands for derivation with respect to an affine parameter which in the case of a massive particle can be taken as the proper time. The resulting equation is

$$\frac{\exp[2\nu(\tilde{r})]}{\left[1 - \alpha - \frac{2\tilde{m}(\tilde{r})}{\tilde{r}}\right]}(\dot{\tilde{r}})^2 + \exp[2\nu(\tilde{r})]\left[\frac{L^2}{(\tilde{r})^2} + 1\right] = E^2 \quad , \tag{9}$$

where E and L are the energy and the angular momentum per unit of mass of the test particle respectively. This equation shows that the radial motion is the same as that of a particle with a "position dependent mass" with energy $E^2/2$ in ordinary one-dimensional, nonrelativistic mechanics moving in the effective potential

$$V_{eff} = \exp[2\nu(\tilde{r})]\left[\frac{L^2}{(\tilde{r})^2} + 1\right]. \tag{10}$$

In Figure 1 we show the effective potential V_{eff} (see eq. (10)) for the deficit angles $\alpha = 0.43$ and $\alpha = 0.125$ respectively and for three values of the angular momentum; we note the presence of a potential well and therefore the existence of stable circular orbits.

The rotation curves (RC) of spiral galaxies are deduced from the red and blue shifts of the emitted radiation by stars moving in "circular orbits" on both sides of the central region [11]. We therefore evaluate the frequency shift for a light signal emitted from a "test star" in circular orbit in the spacetime of a global monopole and received by a static observer at infinity. The two maximum and minimum frequency shifts (associated with a receding or an approaching star respectively) turn out to be:
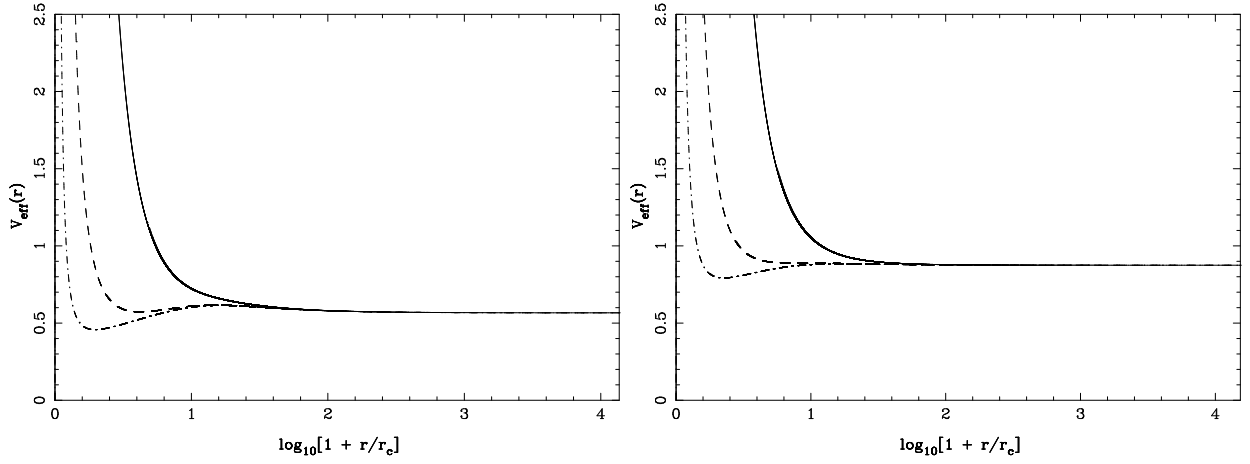
FIG. 1. Functional dependence of the effective potential V_{eff} vs $\tilde{r}$ for the case $\xi = -2$, $\alpha = 0.43$ (left fig.) and $\alpha = 0.125$ (right fig.). For each configuration we show three values of the angular momentum: $L = 4$ (solid line), $L = 1$ (dashed line) and $L = 0.3$ (dash-dotted line). Here $r_c \equiv (\eta\lambda^{1/2})^{-1}$.

$$z_{\pm} = 1 - \left[\frac{(1-\alpha)}{N(\tilde{r})(N(\tilde{r}) - \tilde{r}N(\tilde{r})_{,\tilde{r}})}\right]^{1/2}\left[1 \mp \left(\frac{\tilde{r}N(\tilde{r})_{,\tilde{r}}}{N}\right)^{1/2}\right] \; , \tag{11}$$

with z defined as usual,

$$z \equiv \frac{\Delta\omega}{\omega} = \frac{\omega_R - \omega_E}{\omega_R} \; , \tag{12}$$

where ω_E (ω_R) is the emitted (detected) frequency. We can define $z_D = \frac{1}{2}(z_+ - z_-)$ which would be the quantity that is operationally identified with the velocity $|v(\tilde{r})|$ of the RC of spiral galaxies

$$z_D = \left[\frac{(1-\alpha)}{N(\tilde{r})(N(\tilde{r}) - \tilde{r}N(\tilde{r})_{,\tilde{r}})}\right]^{1/2}\left[\frac{\tilde{r}N(\tilde{r})_{,\tilde{r}}}{N}\right]^{1/2} \; . \tag{13}$$

In Figure 2 we show z_+ (dashed line), z_- (solid line) and z_D (dash-dotted line) as functions of $\tilde{r}$ for the cases $\alpha = 0.43$ (left fig.) and $\alpha = 0.125$ (right fig.) respectively. We note that even for this very simple model the figures that would correspond to the rotation curves contain a relatively "flat region" within the values of r corresponding to the stable orbits (i.e. the behavior of z_D near its maximum).

We are of course not suggesting that this simple model is in any way a realistic candidate for a natural explanation of the phenomena associated with the RC of spiral galaxies. However, the fact that the two main objections against the study of global monopoles in this context can be removed by such a simple modification as the introduction of a nonminimal coupling and the surprising coincidences mentioned at the beginning of this letter, provide very suggestive motivations for pursuing in the analysis of such possibilities. We must, nevertheless, emphasize that monopole-antimonopole pairs annihilate rather efficiently [12] and therefore the viability of this scenario, as applied to the galactic rotation curves, will also depend strongly on the degree of asymmetry in the number of monopoles and antimonopoles in the early universe.

In conclusion we have shown that the qualitative difficulties of the original monopole scenario for explaining the rotation curves of galaxies can be solved by the introduction of nonminimal coupling. The challenge for the future is to look for a specific realization of this more complicated scenario that combines the qualitative success of the latter model with the quantitative coincidences of the former.
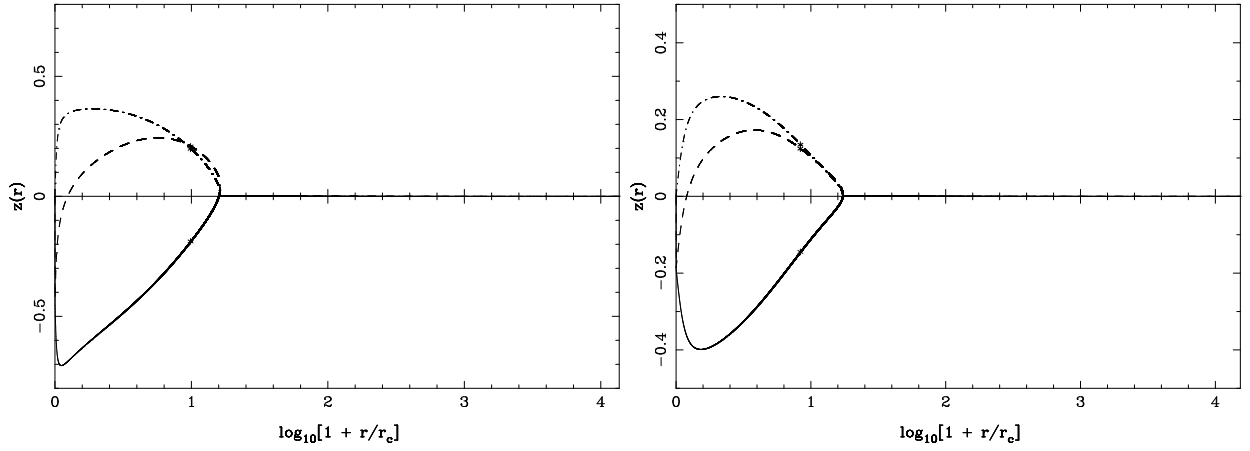
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FIG. 2. Functional dependence of z_+ (dashed line), z_- (solid line) and z_D (dash-dotted line) vs $\tilde{r}$ for circular orbits in the case $\xi = -2$, $\alpha = 0.43$ (left fig.) and $\alpha = 0.125$ (right fig.). The asterisk depicts the location of the radius beyond which the stable circular orbits cease to exist ($r \sim 9\, r_c$).

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